Numerical Simulation of Internal Waves in the Littoral Ocean

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LONG-TERM GOAL

Our long-term goal is to employ numerical simulation to generate accurate predictions of nonhydrostatic internal-tide events, such as large internal waves and solitons, in the coastal areas of the ocean.

OBJECTIVES

Our oceanographic objective is to work collaboratively with oceanographers who have carried out field-scale experiments to quantify the significant wave events triggered by internal tides, including the nonhydrostatic formation of solitons and their evolution.

Our numerical objective is to improve an existing field-scale code so we may simulate Monterey Bay internal waves in three dimensions and time.

APPROACH

Laboratory-scale simulation work has been carried out to examine in detail the physics of breaking internal waves and to test theory against repeatable laboratory experiments done in the Environmental Fluid Mechanics Laboratory. The primary goal of this work was to quantify the mixing efficiency of laboratory-scale breaking interfacial waves in order to parameterize the effects for our larger scale models. The code used to model these laboratory-scale waves employs many of the features of the Casulli code mentioned below. This work is reported in Fringer and Street (2003) as listed in the Publications section.

For simulations of the coastal ocean, we are using the UnTRIM field-scale code [Casulli, 1999 a & b and Casulli and Walters, 2000]. It is a finite-volume, nonhydrostatic and free-surface code that employs an unstructured, staggered-grid. This code uses cells composed of Delaunay triangles in the horizontal plane with layers of uniform [but arbitrary] thickness in the vertical. The triangles allow boundary following in plan form, with a variable grid spacing so that one can concentrate grid points over canyons, etc. The thickness of the layers varies from layer to layer.

In our first attempts to simulate internal waves in Monterey Bay, the tides behaved well, but the internal waves were suppressed. This led to our understanding of the following critical issues:

(a) Low numerical dissipation: By a separate simulation of an internal wave in a tank, we demonstrated that it is the numerical dissipation in UnTRIM that significantly suppresses the internal waves. The numerical dissipation is introduced by the first-order Euler-Lagrangian method (ELM) used in UnTRIM. It appears that schemes with low numerical dissipation must be used in internal wave simulations. The advection scheme used in UnTRIM needed to be modified.

(b) Accurate scalar transport scheme: Similarly an accurate, conservative, and oscillation-free scalar transport scheme was required.

The advection schemes for both momentum and scalars employ ELM, i.e., a traceback from time $t+dt$ to time t is made along a characteristic of the advection terms in the equations. At the intersection of the characteristic and the time t physical space, one must interpolate the values of the relevant variable from the grid points of the mesh to the location of the characteristic intersection. This interpolated value is the value of the advected quantity at time $t+dt$. A significant source of dissipation in UnTRIM is the linear interpolation used. To remove this requires a high-order interpolation scheme on the unstructured grid. We have been using the Kriging interpolation scheme. Kriging is a geostatistical interpolation method formulated directly for irregularly sampled data points. Kriging assumes that a physical phenomenon may be represented by a *spatially* random function. The unknown values can be estimated by a weighted linear combination of the known values at the sampled points. In Kriging, the weights are chosen based on the principle of "best linear unbiased estimation" (BLUE). In addition, Kriging requires the specification of a generalized covariance function, which is a measure of the spatial correlation of the random function. We employ a polynomial covariance function. In addition, we have demonstrated via theory that the use of so-called drift functions can materially improve the behavior of the Kriging. They introduce lower order terms into the Kriging series and remove low-order truncation error in the interpolation. We use Kriging then to interpolate the values of the flow variables (velocities and scalars) and their derivatives on an unstructured grid. A formulation for obtaining derivatives of the variables was derived also because that will be useful in LES formulations using sub-grid-scale models.

WORK COMPLETED:

In the past year, we have:

1. completed preparation of a publication on our laboratory-scale simulations and their comparison to experimental results.

2. conducted accuracy analysis of the Kriging interpolation scheme in the context of finite differencing and made comparisons with the widely used Lagrangian interpolation scheme.

3. developed a high-order advection scheme for momentum by incorporating the Kriging interpolation scheme into the Euler-Lagrangian method.

4. conducted accuracy analysis of the resulting high-order advection scheme and compared it to Euler-Lagrangian method using Lagrangian interpolation scheme.

5. created an accurate and conservative advection scheme for scalar transport based on Kriging.

RESULTS

1. Accuracy analysis of kriging interpolation scheme: The objective of our analysis was to understand the accuracy of the Kriging interpolation scheme in the context of finite differencing. The analysis is conducted by using a Taylor series expansion of the solution to the simple advection equation in one dimension. The results are presented in terms of a general formula for the truncation error. This general formula can be applied to a general interpolation scheme, as well as a specific interpolation scheme, such as the Kriging interpolation scheme and the Lagrangian interpolation scheme, and results in the truncation error for that specific scheme. The conclusions of this work were:

(a) For the Kriging interpolation scheme:

- The order of the leading truncation error term is determined by the order of drift being used. A Kriging interpolation scheme with an m^{th}-order drift can produce accurate interpolation with a leading truncation error proportional to the $(m+1)^{th}$-derivative term $\frac{\partial^{m+1} u}{\partial x^{m+1}}$. For instance, Kriging with a linear drift approximates the 1st-derivative accurately (Figure 1), with the leading truncation error being the 2nd-derivative term (Figure 2). Lower-order terms are interpolated exactly.

- The magnitude of the leading truncation error is determined by the drift, the covariance function, and the number of points being used in the interpolation.

- The minimum number of points required in Kriging is determined by the drift, i.e., for an m^{th}-order drift, $(m+1)$ points is the minimum.

(b) For Lagrangian interpolation:

- The order of the leading truncation error term is determined by the order of polynomial interpolation being used, i.e., an m^{th}-order interpolation results in a leading truncation error proportional to the $(m+1)^{th}$ derivative term $\frac{\partial^{m+1} u}{\partial x^{m+1}}$.

- The magnitude of the leading truncation error is determined by the magnitude of the weights, which are determined by the polynomial interpolation being used.

- Once the order of interpolation is determined, the number of points that can be used is fixed, i.e., for an m^{th}-order interpolation, $(m+1)$ points are required.

(c) The Kriging interpolation scheme is comparable to the Lagrangian interpolation scheme in the context of finite differencing. A comparison between the Kriging interpolation and the Lagrangian interpolation is shown in Figure 2. It is clear that the truncation error term introduces positive diffusion here and so dissipation in the momentum equation. This is much reduced by even a low-order Kriging.

2. High-order advection scheme for momentum: A high-order advection scheme for momentum was developed by forming a high-order Euler Lagrangian method (ELM) using the Kriging interpolation scheme on the unstructured grid. The resulting high-order Kriging-ELM advection scheme has been incorporated into our code. Figure 3 shows the reduction in dissipation when the linear ELM is replaced by Kriging [with quadratic drift] in only the horizontal direction; no other changes were made to the code so that other sources of dissipation remain. Figure 3 shows the evolution of the free surface in a 10 m long by 10 m deep started from rest with a cosine-shaped free surface. While the simulation is actually three-dimensional, there is no cross-tank motion in this case.

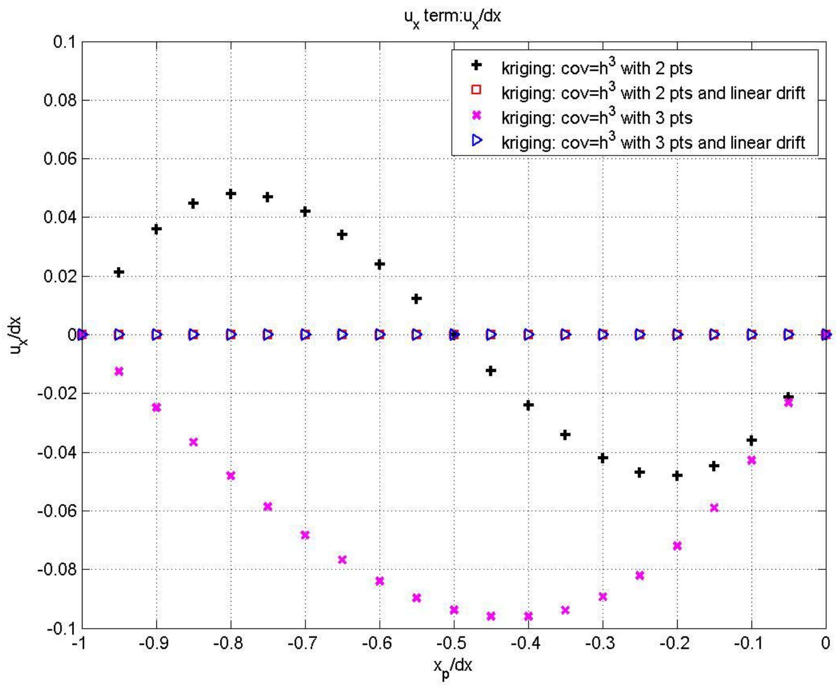

Figure 1. The effect of drift on the accuracy of Kriging interpolation of a variable. The ordinate gives the location along a simple wave component and the abscissa is the normalized [by grid spacing] error in the first derivative's interpolation. The covariance function h is the distance between data and interpolation points; the higher power to which h is raised means greater accuracy.

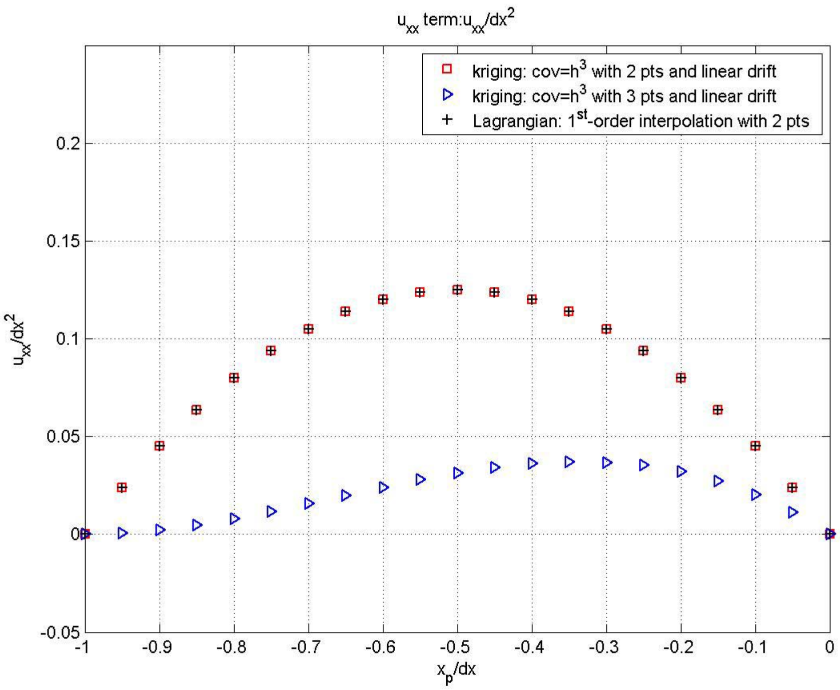

Figure 2. Comparison between Kriging and Lagrangian interpolation. The abscissa is the normalized error in the second derivative of the function being interpolated.

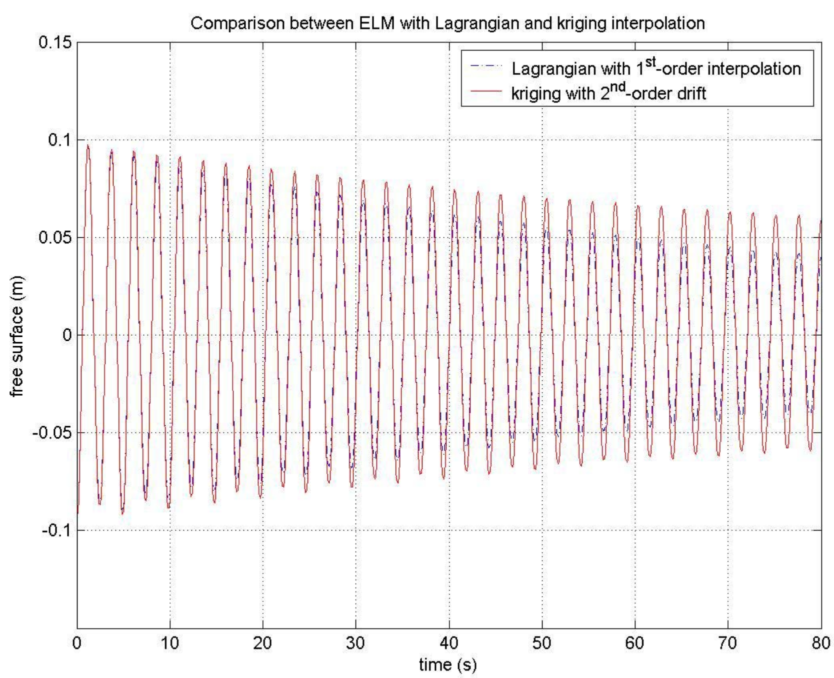

Figure 3. Evolution of the free surface height at one end of a 10 m by 10 m tank of water, comparing Kriging and Lagrangian interpolations. Motion was initiated with a 20-m long cosine wave.

3. Accurate and conservative advection scheme for scalar transport: An advection scheme for scalar transport was developed by using a high-order ELM [Kriging-based] in a finite volume frame. The resulting scalar scheme has been incorporated into our code. Currently, we are working on analyzing the accuracy of the advection scheme for scalar transport and testing the performance of the resulting scheme.

4. Next steps:

(a) Code issues: Our work has been materially slowed by a persistent asymmetry introduced into the sloshing tank simulation by the nonhydrostatic formulation in the original UnTRIM code. Our first priority has been to correct this asymmetry.

(b) Higher-order advection: The Kriging interpolation shows great potential in constructing numerical schemes on an unstructured grid. This is important for coastal ocean modeling since numerical schemes using an unstructured grid offer great flexibility and efficiency in capturing the important physics in coastal oceans. Our work is among the first studies to use Kriging in constructing high-order numerical schemes to study coastal internal waves.

Our preliminary work showed that the accuracy of Kriging is determined by the drift, the covariance function, and the number of points being used in the interpolation. Further study is desired to understand the dependency of the Kriging interpolation accuracy on the above factors. This will lead us to better understand the behavior of the Kriging interpolation scheme and its application in constructing high-order numerical schemes for coastal ocean modeling.

Previous work (Kitanidis, 1997, Le Roux et al., 1997 and 2000) showed that Kriging in general is very computationally-expensive, which is not a desired feature in numerical schemes for large-scale coastal modeling. Further work is needed to understand the correlation between the Kriging accuracy and the Kriging efficiency. This will help us implement the Kriging scheme in an effective way. In parallel we will examine the Eulerian advection scheme being implemented in the SUNTANS code by Fringer under ONR Grant N00014-02-1-0204 and NSF Grant OCE-0113111. Our initial test cases will be free-surface and internal-wave sloshing in a large tank as described above. We expect to generate a definitive comparison of these two schemes. This will aid all the projects.

IMPACT/APPLICATIONS

This unstructured grid code with accurate advection schemes for momentum and scalars will be able to generate insights to internal wave behavior over complex bathymetry with additional resolution easily and smoothly added in regions of significant variations in bathymetry and/or physical behavior of t he waves.

RELATED PROJECTS

Work under ONR Grant N00014-02-1-0204 and NSF Grant OCE-0113111 will provide an advanced momentum and energy conserving Eulerian advection formulation to test versus the Kriging approach.

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PUBLICATIONS

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